MATH 1650: SECTION 9.2: SUMMATION NOTATION

In this section, we introduce and practice notation for adding parts of sequences. For example,

$$\begin{aligned}\sum_{n=3}^{6}(2n-1) &= (2(3)-1)+(2(4)-1)+(2(5)-1)+(2(6)-1)\\ &= 5+7+9+11\\ &= 32\end{aligned}$$

The variable 'n' is called the **index** of the summation and is considered a 'dummy variable' in the sense that it may be changed to any letter without affecting the value of the summation. For instance,

$$\sum_{n=3}^{6}(2n-1)=\sum_{k=3}^{6}(2k-1)=\sum_{j=3}^{6}(2j-1)$$

EXAMPLE: Write out the following sums:

1. $\displaystyle\sum_{k=1}^{4}\frac{13}{100^k}$

2. $\displaystyle\sum_{n=0}^{4}\frac{n!}{2}$

   HINT: Recall: $n! = n(n-1)\cdots(1)$ with $0! = 1$.

3. $\displaystyle\sum_{n=1}^{5}\frac{(-1)^{n+1}}{n}(x-1)^n$

EXAMPLE: Write the following sums using summation notation.

4. $1 + 3 + 5 + \ldots + 117$

5. $1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + - \ldots + \dfrac{1}{117}$

6. $0.9 + 0.09 + 0.009 + \ldots 0.\underbrace{0 \cdots 0}_{n-1 \text{ zeros}}9$

PROPERTIES OF SUMMATION NOTATION:

- **Sum and Difference Property:** $\displaystyle\sum_{n=m}^{p}(a_n \pm b_n) = \sum_{n=m}^{p} a_n \pm \sum_{n=m}^{p} b_n$
- **Distributive Property:** $\displaystyle\sum_{n=m}^{p} c\, a_n = c \sum_{n=m}^{p} a_n$, for any real number c.
- **Additive Index Property:** $\displaystyle\sum_{n=m}^{j} a_n + \sum_{n=j+1}^{p} a_n = \sum_{n=m}^{p} a_n$, for any natural number $m \le j < j+1 \le p$.
- **Re-indexing:** $\displaystyle\sum_{n=m}^{p} a_n = \sum_{n=m+r}^{p+r} a_{n-r}$, for any integer r.

EXAMPLE: Use properties of sums to help you work the following problems:

7. If $\displaystyle\sum_{n=2}^{50}(a_n - 3b_n) = 17$ and $\displaystyle\sum_{n=2}^{50} a_n = 10$, find $\displaystyle\sum_{n=2}^{50} b_n$.

8. If $\displaystyle\sum_{n=1}^{20} a_n = -3$ and $\displaystyle\sum_{n=1}^{21} a_n = 7$, find a_{21}.

9. Rewrite the sum so the index starts at 0: $\displaystyle\sum_{n=2}^{437} n(n-1)x^{n-2}$

SUMS OF ARITHMETIC AND GEOMETRIC SEQUENCES:

- The sum S of the first n terms of an arithmetic sequence $a_k = a + (k-1)d$ for $k \geq 1$ is

$$S = \sum_{k=1}^{n} a_k = n\left(\frac{a_1 + a_n}{2}\right) = \frac{n}{2}(2a + (n-1)d)$$

- The sum S of the first n terms of a geometric sequence $a_k = ar^{k-1}$ for $k \geq 1$ is

1. $S = \sum_{k=1}^{n} a_k = \dfrac{a_1 - a_{n+1}}{1-r} = a\left(\dfrac{1-r^n}{1-r}\right)$, if $r \neq 1$.
2. $S = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} a = na$, if $r = 1$.

EXAMPLE:

10. Find the sum: $1 + 3 + 5 + \ldots + 117$

11. Find a formula for the sum $\sum_{k=1}^{n} k$

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12. Suppose you start saving 1 penny on February 1st, then 2 pennies on February 2nd, 4 pennies on February 3rd, and so on - saving double the number of pennies as the day before. How much money will you have altogether after a week? Two weeks? Three weeks? A month?

FUTURE VALUE OF AN ORDINARY ANNUITY: Suppose an annuity offers an annual interest rate r compounded n times per year. Let $i = \frac{r}{n}$ be the interest rate per compounding period. If a deposit P is made at the end of each compounding period, the amount A in the account after t years is given by

$$A = \frac{P\left((1+i)^{nt} - 1\right)}{i}$$

EXAMPLE: An ordinary annuity offers a 6% annual interest rate, compounded monthly.

13. If monthly payments of \$50 are made, find the value of the annuity in 30 years.

14. How many years will it take for the annuity to grow to \$100,000?

HOMEWORK: Section 9.2: 1 - 43 odd.